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Brane dynamics in background fluxes and non-commutative geometry

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Abstract: Branes in non-trivial backgrounds are expected to exhibit interesting dynamical properties. We use the boundary conformal field theory approach to study branes in a curved background with non-vanishing Neveu-Schwarz 3-form field strength. For branes on an S^3, the low-energy effective action is computed to leading order in the string tension. It turns out to be a field theory on a non-commutative "fuzzy 2-sphere" which consists of a Yang-Mills and a Chern-Simons term. We find a certain set of classical solutions that have no analogue for flat branes in euclidean space. These solutions show, in particular, how a spherical brane can arise as bound state from a stack of D0-branes.

Keywords: Conformal Field Models in String Theory, Brane Dynamics in Gauge Theories, Non-Commutative Geometry, Boundary Quantum Field Theory.

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1. Introduction

Many important properties of D-branes can be understood in terms of open strings attached to the world-volume of the brane. At sufficiently low energies, the behavior of these open strings may be described through an effective field theory of the light modes from the open string spectrum. Typically, these field theories involve vector fields propagating on the brane along with a number of scalar fields that describe fluctuations of the brane in the transverse direction.

It was discovered more recently that the presence of a background B-field causes the world-volume of the brane to become non commutative. The initial observation for branes on a 2-torus [1, 2] was clarified and generalized in a number of papers [3]–[6]. One can see this non-commutative geometry of branes in their effective action. In fact, it naturally becomes a field theory on a non-commutative space so that the non-commutativity of the world-volume has direct influence on the dynamics of the brane.

Much of the existing work in this direction has been devoted to branes in flat space. A perturbative analysis along the lines of [5], on the other hand, shows that the quantization of world-volume geometries should be a much more general phenomenon which persists in the case of curved backgrounds. In fact, quantization of world-volumes appears to be inevitable in curved backgrounds because the string equation of motion enforce a non-vanishing NS 3-form field strength H whenever the underlying space is not Ricci-flat. Since a non-zero field strength cannot possibly possess a vanishing 2-form potential B, branes in curved space are necessarily equipped with a non-zero B-field and hence they are quantized.

The first rigorous analysis of brane non-commutativity in a curved space, namely on a 3-sphere, was presented in [7], using the world-sheet approach to D-branes. In that work, we gave a prescription to extract non-commutative brane world-volumes from the data of a boundary CFT. This definition builds on the closed string ideas of [8] (see also [9] for examples) and is independent of notions from classical geometry, thus providing further evidence that brane non-commutativity is a general feature.

An exact conformal field theory description is available for certain maximally symmetric branes on an S^3. It was shown in [10] that, classically, these branes are wrapped on conjugacy classes of $\mathrm{SU}(2) \cong S^3$. The generic classes are 2-spheres in S^3, and due to the B-field they are quantized [7] such that the algebra of "functions on the brane" is a full matrix algebra or some (mildly non-associative) q-deformation thereof, depending on the size of the 3-sphere in which the branes are embedded. The size of the matrix algebra is determined by the radius of branes themselves. These world-volume algebras are also known as "fuzzy spheres", see [11, 12] and references therein.

As these fuzzy spheres are now embedded into a full string theory with all its dynamical degrees of freedom, we may wonder about the field theory that describes the light open string modes living on the fuzzy sphere. Field theories with the expected field content have been described in the non-commutative geometry literature [12]–[16]. In particular, Klimčík [16] has suggested a family of actions which are arbitrary linear combinations of non-commutative Yang-Mills and Chern-Simons terms. The string theory effective action is of this type but selects a particular member of this family for which the mass terms present in the non-commutative Yang-Mills and Chern-Simons actions cancel each other.

We will see that our field theories on the fuzzy sphere possess interesting classical solutions some of which have no analogue in effective field theories on flat branes. They describe stacks of spherical branes that evolve into superpositions of spherical branes with different radii. In particular, a stack of N D0-branes (i.e. N spheres of vanishing radius) can condense into a single D2-brane of radius $r \sim N$. This dynamical generation of new world-volume dimensions occurs at the expense of a partial breaking the gauge symmetry.

The formation of extended objects from smaller constituents has of course been discussed extensively in the context of M-theory (see e.g. [17] and references therein) and also for branes in constant RR-background fluxes [18]. While the latter work takes the Born-Infeld action as a starting point, our analysis is based on an exact open string background given by a boundary CFT. In this way, we are also able to show that the effects in question are inherited from the brane's non-commutativity.

This requires, however, to understand the brane dynamics beyond small fluctuation. The latter were studied recently in [19, 20] in order to establish the stability of branes on group manifolds. Many of the statements we will make below can easily be extended to other group manifolds, too. We restrict ourselves to the group SU(2) for

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simplicity and because the background SU(2) $\cong S^3$ is also an important ingredient of the CFT formulation of the Neveu-Schwarz 5-brane, see e.g. [21], such that our findings may be useful to study the geometry of D-branes in the presence of a stack of 5-branes. Similarly, our SU(2) WZW results are applicable in the study of branes on an $AdS_3 \times S^3$ string background, see e.g. [22].

The paper is organized as follows: In the next section we will briefly list a few facts about branes, B-fields and non-commutative geometry of branes in flat space, including a few important formulas that are rather convenient to prepare the ground for the following less standard discussion. Section 3 is mainly a review of the results in [10, 7]. It contains all the results we shall need for our computation of the effective action, which is carried out in section 4. For simplicity, this will be illustrated in the bosonic theory first, but we will sketch how to extend the calculation to full supersymmetric string backgrounds like an NS 5-brane. It will turn out that the (bosonic part of the) low-energy effective action is the same as for bosonic strings on S^3. Section 5 contains a detailed analysis of classical solutions and their interpretation. Finally, we end with conclusions and some open problems.

2. Flat branes and non-commutative gauge theory

Before we show how to calculate low-energy effective actions for branes wrapping an $S^2 \subset S^3$, let us briefly review a few important steps of the analogous calculation in the standard case of branes in flat n-dimensional euclidean space $\mathbb{R}^n$, or on a flat torus $\mathbb{T}^n$. Consider a D-brane which is localized along a p-dimensional hyper-plane V_p in the target, with tangent space TV_p. The conformal field theory associated with such a euclidean D-brane is defined on the upper half of the complex plane. It contains an n-component free bosonic field $X = (X^\mu(z, \bar{z}))$, $\mu = 1, \ldots, n$, subject to Neumann boundary conditions in the directions along TV_p and Dirichlet boundary conditions for components perpendicular to the world-volume of the brane. From the free bosons, one may obtain various new fields, in particular the vertex operators for the open string tachyons

$$V_k(x) = :\exp(ikX(x)): \quad \text{for all } k \in TV_p\,,$$

which can be inserted at any point x on the real line. These operators correspond to the lightest states $|k\rangle$ within each sector of fixed momentum k. One can think of them as being assigned to the eigenfunctions $e_k = \exp(iku)$ of the momentum operator on the world-volume V_p of the brane. To make this more explicit we will use the notation $V[e_k](x) = V_k(x)$.

When there is a magnetic field $B = (B_{\mu\nu})$ on the brane, the operator product expansion (OPE) of these U(1)-primaries reads

$$V[e_{k_1}](x_1)V[e_{k_2}](x_2) = \frac{1}{(x_1 - x_2)^{2\alpha' k_1 \cdot k_2}} e^{-i\frac{\pi\alpha'}{2}\, k_1^{\mathrm{t}} \Theta k_2}\; V[e_{k_1+k_2}](x_2) + \cdots\,,$$

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up to contributions involving descendant fields. In writing such OPEs of boundary operators we assume throughout the whole paper that their insertion points are ordered $x_1 > x_2$. The product $k_1 \cdot k_2 = G^{\mu\nu} k_{1,\mu}\, k_{2,\nu}$ is to be taken with the inverse of the open string metric G rather than of the original closed string metric $g_{\mu\nu}$. $G^{\mu\nu}$ can be read off from the standard expression $h = \alpha' G^{\mu\nu} k_\mu k_\nu = 1/2\, G^{\mu\nu}(\sqrt{2\alpha'}k_\nu)(\sqrt{2\alpha'}k_\mu)$ for the scaling dimension of the boundary primary fields $V[e_k]$. The relation of Θ and G to the closed string parameters B and $g = 1$ is given by [5]

$$\Theta = \frac{2}{B - B^{-1}} \qquad \text{and} \qquad G = \frac{1}{1 - B^2}\,. \tag{2.1}$$

In a particular limit ("zero slope limit" [6]) of the theory for which $\alpha' G^{\mu\nu}$ becomes zero, the boundary fields have vanishing conformal dimension and the operator product expansions are no longer singular. Thus we obtain

$$V[e_{k_1}](x_1)\, V[e_{k_2}](x_2) = V[e_{k_1} * e_{k_2}](x_2)\,.$$

The product $e_{k_1} * e_{k_2} = \exp(-i\frac{\pi\alpha'}{2} k_1^{\mathrm{t}} \Theta k_2) e_{k_1} e_{k_2}$ is the Moyal-Weyl product of functions on V_p (see [5, 6] for details). We can pass to a more universal form that does not make reference to the basis e_k in the space of functions: if we introduce $V[f] = \sum \hat{f}_k V[e_k]$ with $\hat{f}_k$ being the Fourier coefficients of f, we obtain

$$V[f](x_1) V[g](x_2) = V[f * g](x_2)\,.$$

This formula holds for arbitrary functions f, g as long as we are in the zero-slope limit. In order to compute the low-energy effective action, we need two further operator products involving the boundary current $j^\mu(x)$, namely

$$j^\mu(x_1) j^\nu(x_2) = \frac{\alpha'}{2} \frac{G^{\mu\nu}}{(x_1 - x_2)^2} + \cdots\,, \tag{2.2}$$

$$j^\mu(x_1) V[f](x_2) = \frac{\alpha' G^{\mu\nu}}{x_1 - x_2} V[-i\partial_\nu f](x_2) + \cdots\,. \tag{2.3}$$

With the help of these equations, the n-th-order terms for massless fields in the LEEA are obtained from an n-point function of operators $: j^\mu V[A_\mu] : (x)$ where A_μ is the (possibly non abelian) gauge field, i.e. $A_\mu(u)$ are functions on the world-volume of the brane, which take values in $\mathrm{Mat}(N)$ for a stack of N-identical branes. In the zero-slope limit, the resulting action is given by a gauge theory on the non-commutative plane or torus [1, 6],

$$\mathcal{S}(A)_{\text{flat}} = \frac{1}{4} \int d^p u F_{\mu\nu} * F^{\mu\nu}\,,$$

where $F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu * A_\nu]$ and where $*$ is the Moyal-Weyl product; the integration extends over the world-volume of the brane.

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3. Branes on S^3 and fuzzy spheres

Strings moving on a 3-sphere are described by the SU(2) WZW model at level k. Here, the radius R of S^3 is related to the level by $R \sim \sqrt{\alpha'\mathsf{k}}$. Because S^3 is curved, the string equations of motion enforce the presence of a constant NS 3-form field strength $H \sim \Omega/\sqrt{\alpha'\mathsf{k}}$, where Ω is the volume form on the unit sphere.

The worldsheet swept out by an open string in S^3 is parametrized by a map $g : \mathrm{H} \to \mathrm{SU}(2)$ from the upper half-plane H into the group manifold $\mathrm{SU}(2) \cong S^3$. From this field g one obtains Lie algebra valued chiral currents

$$J(z) = -\mathsf{k}(\partial g)g^{-1}\,, \qquad \bar{J}(\bar{z}) = \mathsf{k}g^{-1}\bar{\partial}g$$

as usual. There exists an exact conformal field theory description for maximally symmetric D-branes on SU(2), which are characterized by the gluing condition $J(z) = \bar{J}(\bar{z})$ along the boundary $z = \bar{z}$. The geometry of the associated branes is encoded in these gluing conditions [10]. First of all, they force the endpoints of open strings to stay on conjugacy classes of SU(2). The generic such classes are 2-spheres, except for two cases where the classes degenerate into the single points e resp. $-e$, where e is the unit element of SU(2). Moreover, the condition $J = \bar{J}$ selects a particular two form potential B ("B-field") on the brane, which is given by

$$B = \frac{\mathrm{Ad}(g) + 1}{\mathrm{Ad}(g) - 1}\,.$$

Here $\mathrm{Ad}(g)$ denotes the adjoint action of the SU(2) on the tangent space of the brane. The existence of a non-vanishing B-field causes the brane world-volume to become non commutative. From the previous analysis of branes in flat space we know that it is the antisymmetric tensor Θ rather than the B-field itself (or its inverse) that describes the semi-classical limit of the brane's non-commutative world-volume algebra. Θ is computed from B by means of the formula (2.1), leading to

$$\Theta = \frac{2}{B - B^{-1}} = \frac{1}{2}\left(\mathrm{Ad}(g) - \mathrm{Ad}(g)^{-1}\right). \tag{3.1}$$

The semi-classical extension of the above analysis shows that, for fixed gluing conditions, only a finite number of SU(2) conjugacy classes satisfy a Dirac-type flux quantization condition [10, 23]. These "integer" conjugacy classes are the two points e and $-e$ along with $\mathsf{k}-1$ of the spherical conjugacy classes, namely those passing through the points $\mathrm{diag}(\exp(2\pi i\alpha/\mathsf{k}), \exp(-2\pi i\alpha/\mathsf{k}))$ for $\alpha = 1/2, \ldots, (\mathsf{k}-1)/2$.

Our discussion of brane dynamics in the next section will be based upon a computation of the leading non-vanishing terms of the low-energy effective action around the point $1/\alpha' k = 0$. For this reason, let us briefly look at the classical geometry in the limit $\alpha'\mathsf{k} \to \infty$ in which the 3-sphere grows and approaches flat 3-space. If one parametrizes g by X taking values in the Lie algebra su(2), such that $g \approx 1 - X$,

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the expression for Θ becomes $\Theta = \mathrm{ad}(X)$. This is the standard Kirillov bi-vector on the spheres in the algebra $\mathrm{su}(2) = \mathbb{R}^3$. Consequently, the geometry of the limiting theory $\alpha'\mathsf{k} = \infty$ is very close to the well-known situation of flat branes in a flat background with constant B-field, and we expect that the world-volume algebras of our branes in the WZW model will be quantizations of 2-spheres equipped with their standard Poisson structure. According to Kirillov's theory, all "integer" 2-spheres are quantizable and in one-to-one correspondence with irreducible representations of SU(2). Kirillov's theory implies that "functions" on the quantized 2-spheres form matrix algebras $\mathrm{Mat}(2\alpha+1)$ of size $2\alpha+1$ where α can be any of the numbers $\alpha = 0, 1/2, 1, \ldots$ In this way we have identified our branes with representations of SU(2) such that the brane's label α — which is proportional to its radius — denotes the spin of the associated representation.

In addition to the geometric picture developed so far, we will now list a number of exact results from the boundary conformal field theory description [24] of the associated quantum theory of open strings. Their derivation is explained in [7]. The chiral fields of the SU(2) WZW model form an affine Kac-Moody algebra denoted by $\widehat{\mathrm{SU}}(2)_{\mathsf{k}}$. It is generated by fields $J_a(z), a = 1, 2, 3$, which possess the following operator product expansions:

$$J_a(x_1)J_b(x_2) = \frac{\mathsf{k}}{2}\frac{\delta_{ab}}{(x_1-x_2)^2} + \frac{if_{ab}{}^c}{(x_1-x_2)}J_c(y) + \cdots , \tag{3.2}$$

where f is the Levi-Civita tensor. The open string vertex operators of the D-brane theories we are dealing with are organized in representations of this symmetry algebra. For the brane with label $\alpha = 0, 1/2, \ldots, \mathsf{k}/2$, there appear only finitely many such representations with integer angular momentum between $J = 0$ and $J = \min(2\alpha, \mathsf{k} - 2\alpha')$. Each representation J contains ground states of lowest possible energy, which form $(2J+1)$-dimensional multiplets spanned by some basis vectors Y^J_m with $|m| < J$. To these states we assign vertex operators $V[Y^J_m](x)$. They are similar to the tachyon vertex operators $V[e_k]$ for open strings in flat space. While the latter are associated with eigenfunctions of linear momentum k, our vertex operators $V[Y^J_m]$ carry definite angular momentum, and one may think of the elements Y^J_m as spherical harmonics on the 2-sphere that forms the classical world-volume of the brane α. Because of the quantization, there appears a cutoff on the angular momentum, i.e. J is restricted by $J \le \min(2\alpha, \mathsf{k} - 2\alpha')$. The operator product expansion of these open string operators was found to be of the form [25, 7]

$$\begin{aligned} &V\left[Y^I_i\right](x_1)V\left[Y^J_j\right](x_2) = \\ &\quad = \sum\nolimits_{K,k} x_{12}^{h_K - h_I - h_J} \begin{bmatrix} I & J & K \\ i & j & k \end{bmatrix} F_{\alpha K} \begin{bmatrix} J & I \\ \alpha & \alpha \end{bmatrix} V\left[Y^K_k\right](x_2) + \cdots , \end{aligned} \tag{3.3}$$

where h_J is the conformal dimension of $V[Y^J]$, $\left[\begin{smallmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{smallmatrix}\right]$ denote the Clebsch-Gordan coefficients of the group SU(2), and F stands for the fusing matrix of the WZW-

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model. In the limit $\mathsf{k} \to \infty$, the fusing matrix elements approach the $6J$ symbols of the classical Lie algebra su(2). At the same time the conformal dimensions $h_J = J(J+1)/(\mathsf{k}+2)$ tend to zero so that the OPE (3.3) of boundary fields becomes regular as in a topological model.

For the limiting theory there exists a much more elegant way of writing the operator product expansion (3.3). To see this observe that SU(2) acts on the space of $(2\alpha+1)\times(2\alpha+1)$-matrices by conjugation with group elements evaluated in the $(2\alpha+1)$-dimensional representation. Under this action, the $(2\alpha+1)^2$-dimensional space $\mathrm{Mat}(2\alpha+1)$ decomposes into a finite set of SU(2) modules of spin $J = 0, 1, \ldots, 2\alpha$. This is precisely the list of representations which appears in the open string theory of our brane α when the level is sent to infinity. Hence, we can identify the ground states $|Y_m^J\rangle$ with matrices $\mathrm{Y}_m^J \in \mathrm{Mat}(2\alpha+1)$. It is a remarkable feature of this identification that the multiplication of the matrices Y_i^I and Y_j^J turns out to encode the information about the operator products in our conformal field theory,

$$V\left[\mathrm{Y}_i^I\right](x_1) V\left[\mathrm{Y}_j^J\right](x_2) = V\left[\mathrm{Y}_i^I * \mathrm{Y}_j^J\right](x_2)\,.$$

The emergence of the matrix product $*$ in this relation was expected from our geometric analysis above. It shows that the world-volumes of branes in S^3 become fuzzy 2-spheres in the quantum theory. We can now rewrite the operator product in a way which does no longer refer to a particular choice of a basis. In fact, for an arbitrary matrix $\mathsf{A} \in \mathrm{Mat}(2\alpha+1)$ with $\mathsf{A} = \sum a_{Jm} \mathrm{Y}_m^J$ we introduce $V[\mathsf{A}] = \sum a_{Jm} V[\mathrm{Y}_m^J]$ and obtain

$$V[\mathsf{A}_1](x_1) V[\mathsf{A}_2](x_2) = V[\mathsf{A}_1 * \mathsf{A}_2](x_2) \tag{3.4}$$

for all $\mathsf{A}_1, \mathsf{A}_2 \in \mathrm{Mat}(2\alpha+1)$. This product allows us to compute arbitrary correlations functions of such vertex operators,

$$\langle V[\mathsf{A}_1](x_1) V[\mathsf{A}_2](x_2) \cdots V[\mathsf{A}_n](x_n) \rangle = \mathrm{tr}(\mathsf{A}_1 * \mathsf{A}_2 * \cdots * \mathsf{A}_n)\,. \tag{3.5}$$

The trace appears because the vacuum expectation value is SU(2) invariant and the trace maps matrices to their SU(2)-invariant component.

Let us close this section on the conformal field theory of branes on S^3 by displaying a further OPE, namely the one that involves a current J^a along with the vertex operator $V[\mathrm{Y}_j^J]$,

$$J_a(x_1)\; V\left[\mathrm{Y}_j^J\right](x_2) = \frac{1}{x_1 - x_2}\; V\left[L_a \mathrm{Y}_j^J\right](x_2) + \cdots \quad \text{with} \quad L_a \mathsf{A} := \left[\mathrm{Y}_a^1 \stackrel{*}{,} \mathsf{A}\right] \tag{3.6}$$

for all matrices $\mathsf{A} \in \mathrm{Mat}(2\alpha+1)$. The operator L_a expresses the action of the angular momentum on spherical harmonics.

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4. Low-energy effective field theory

With the previous description of the world-sheet theory as a firm basis, we want to address the main aim of the paper, namely the construction of the effective action that describes D-branes wrapped on an $S^2 \subset S^3$ at low energies. The 3-spheres, or rather the SU(2) WZW models, occur as internal spaces in (super) string backgrounds like $AdS_3 \times S^3 \times T^4$ or in the world-sheet theory for the background with $\mathsf{k}-2$ Neveu-Schwarz 5-branes, which provide a source for $\mathsf{k}-2$ units of NS 3-form flux through a 3-sphere surrounding their (5+1)-dimensional world-volume. The CFT description of such an NS 5-brane background involves a tensor product of 5+1 free bosons, of a linear dilaton and of an SU(2) WZW model [21]. For our purposes, it will be sufficient to approximate the "external" part M^7 of these targets by a flat spacetime. At the moment, we furthermore restrict ourselves to bosonic strings, but it will turn out that the results for the low-energy effective action carry over the supersymmetric case.

Thus we consider stacks of N Dp-branes which span a certain flat world-volume in $\mathbb{R}^7$ and wrap a (fuzzy) 2-sphere with label α in the S^3-component of spacetime. The massless field of this brane configuration fall into two groups which we want to discuss separately.

First there are the vector bosons and scalars which are associated with the seven external directions transverse to the S^3. When we compute their contribution to the action, we have to use vertex operators $: j^\mu V[A_\mu] V[\mathsf{A}] : (x)$. Here, $\mu = 0, \ldots, 6$ labels the coordinates transverse to the S^3, and A_μ is either a U(N) gauge or a scalar field on the brane depending on whether the direction of μ is parallel or perpendicular to the brane. The third factor $V[\mathsf{A}]$ in the vertex operator corresponds to the ground states for the brane on S^3, which means that A is a matrix of size $2\alpha + 1$. The actual computation of the associated terms in the effective action proceeds exactly as for branes in flat space, except for the contribution from ground states in the WZW factor of the theory. But the evaluation of the latter is completely independent of the first seven directions, and our formula (3.5) shows that the WZW model contributes precisely in the same way as usual Chan-Paton factors do. Hence, the effective action for the fields A_μ appears as an ordinary non-abelian Yang-Mills-Higgs theory on a D$(p-2)$-brane. The effect of the two dimensions which wrap the $S^2 \subset S^3$ is to replace U(N) by the larger gauge group U($N\,(2\alpha+1)$). In particular, non-abelian gauge theories on the world-volume of a single brane appear for $\alpha > 0$.

The discussion of the remaining three fields that come with the S^3 (scalars viewed from the external part of the brane world-volume) requires a lot more effort. To begin with, let us rescale the currents of the WZW model so as to bring their operator product expansions in a form similar to eqs. (2.2) and (2.3). As in the flat case, we extract the inverse of the open string metric from the conformal dimensions, here given by $h_J = J(J+1)/(\mathsf{k}+2)$. To leading order in $1/\mathsf{k}$, this yields the metric

$$G^{ab} = \gamma \delta^{ab} \quad \text{with} \quad \gamma = 2/\mathsf{k}\,. \tag{4.1}$$

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For the Schwinger term in the OPE of WZW currents J_a to resemble eq. (2.2), we introduce $j^a = (\sqrt{2\alpha'}/\mathsf{k})J_a$. The operator product expansions of these rescaled currents are

$$j^a(x_1)j^b(x_2) = \frac{\alpha'}{2}\frac{G^{ab}}{(x_1-x_2)^2} + \alpha'\frac{i\mathsf{f}\,{}^{ab}_{c}}{(x_1-x_2)}j^c(y) + \cdots\ , \tag{4.2}$$

$$j^a(x_1)V[\mathsf{A}](x_2) = \frac{\alpha' G^{ab}}{x_1-x_2}V[\mathsf{L}_b\mathsf{A}](x_2) + \cdots\,, \tag{4.3}$$

where we also use a rescaled angular momentum operator $\mathsf{L}_a = L_a/\sqrt{2\alpha'}$. The structure constants f in the first line are obtained from the structure constants f of the classical Lie algebra su(2) through $\mathsf{f}_{ab}{}^{c} = f_{ab}{}^{c}/\sqrt{2\alpha'}$. Here and in the following, indices $a, b, c, \ldots$ on the objects L and f are raised and lowered with the open string metric G. In particular, this implies that $\mathsf{f}^{ab}{}_{c} = 2f^{ab}{}_{c}/(\sqrt{2\alpha'}\mathsf{k})$.

Let us focus on the WZW boundary fields $:j^aV[\mathsf{A}_a]:(x)$ where $\mathsf{A}_a = \sum a_a^{Jm}\mathrm{Y}^J_m$ is an arbitrary set of matrices subject to the physical state condition $\mathsf{L}^c\mathsf{A}_c = 0$; summation over the three directions $a = 1, 2, 3$ tangent to the 3-sphere is understood. For simplicity, we shall omit the simple factors $V[f]$ that come with the seven transverse directions throughout most of our discussion. They are taken into account if we consider A_a as matrix valued functions on the world-volume of the brane rather than constant matrices. A_a may be regarded as three massless bosons that live on our fuzzy sphere (or the $(p+1)$-dimensional world-volume of the brane). In the limit $\mathsf{k}\to\infty$ they can also be viewed as massless fields depending on the $(p-1)$-brane coordinates which are transverse to the S^3 because the conformal dimensions h_I of the ground states $|\mathrm{Y}^I_i\rangle$ vanish as we send the level to infinity. When we want to describe a stack of N of our α-branes, the coefficients a_a^{Jm} of the fields A_a have to be $N\times N$-matrices and hence $\mathsf{A}_a \in \mathrm{Mat}(N)\otimes\mathrm{Mat}(2\alpha+1)$. We stress, however, that the operators L_a act exclusively on the second-tensor factor and not on the Chan-Paton degrees of freedom.

We are now prepared to compute the effective action for the three fields A_a to leading order in α'. As in the case of flat branes it turns out that only 3- and 4-point functions contribute to this order of the open string scattering amplitudes. For the 3-point function one finds the following expression:

$$\begin{aligned}
&\big\langle :j^aV[\mathsf{A}_a]:(x_1):j^bV[\mathsf{A}_b]:(x_2):j^cV[\mathsf{A}_c]:(x_3)\big\rangle = \\
&\quad= \frac{1}{2}\frac{(\alpha')^2}{x_{23}x_{12}x_{31}}\bigg[\frac{x_{23}}{x_{12}}G^{ab}\mathsf{L}^{c,(1)} - \frac{x_{31}}{x_{12}}G^{ab}\mathsf{L}^{c,(2)} + \frac{x_{31}}{x_{23}}G^{bc}\mathsf{L}^{a,(2)} - \frac{x_{12}}{x_{23}}G^{bc}\mathsf{L}^{a,(3)} + \\
&\qquad\qquad + \frac{x_{12}}{x_{31}}G^{ac}\mathsf{L}^{b,(3)} - \frac{x_{23}}{x_{31}}G^{ac}\mathsf{L}^{b,(1)} - i\mathsf{f}^{\,abc}\bigg]\times \\
&\qquad\times\langle V[\mathsf{A}_a]V[\mathsf{A}_b]V[\mathsf{A}_c]\rangle + O\left((\alpha')^3\right).
\end{aligned} \tag{4.4}$$

Here, the superscript $^{(j)}$ means that the operator acts on the argument of the j-th vertex operator in the correlation function. The terms involving L arise from

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contracting two currents into a Schwinger term and acting on one of the primary fields with the third current. The structure constants f^{abc} appear upon contraction of two currents into the second term of eq. (4.2) and subsequent contraction of the resulting current with the third current in the correlator. In the bosonic situation, there are higher order terms which we did not spell out because they will not be needed below.

From formula (4.4), we recover the corresponding expression for the flat space theory by the replacements $\mathsf{L}^a \to -i\partial^a$ and, more importantly, $\mathsf{f}^{abc} \to 0$. The 3-point functions feel the non-abelian nature of the theory through the terms that involve the Lie algebra structure constants. In the 4-point functions, such terms are suppressed by at least one order of α', and the leading $O((\alpha')^2)$ contributions come from pairwise contractions of two currents into a Schwinger term. They are identical to the corresponding terms in the case of flat D-branes except for the replacement $\mathsf{L}^a \to -i\partial^a$.

Before we can combine this information into some effective action, we need to consider the mass terms of the theory. Recall that the mass of open string modes is determined from the conformal dimensions in the internal sector of the theory through $M^2 = (h-1)/\alpha'$. The modes we consider have conformal dimension $h = h_J + 1$ so that, to leading order in $1/\mathsf{k}$,

$$M^2 = \frac{1}{\alpha'} h_J = \frac{J(J+1)}{\alpha' \mathsf{k}} \,. \tag{4.5}$$

This relation shows that it is the combined limit $\alpha' \to 0$ and $\alpha' \mathsf{k} \to \infty$ that is relevant for our low-energy considerations. The values in (4.5) constitute the spectrum of the operator $2/\mathsf{k} \sum_a \mathsf{L}_a \mathsf{L}_a = \mathsf{L}_a \mathsf{L}^a$ so that the formula for M^2 results in the following mass term for the lagrangian of our theory

$$\mathcal{S}_0 = \frac{1}{2} \operatorname{tr} \left(\mathsf{A}_a * \mathsf{L}_b \mathsf{L}^b \mathsf{A}^a \right) = -\frac{1}{2} \operatorname{tr} \left(\mathsf{L}_b \mathsf{A}_a * \mathsf{L}^b \mathsf{A}^a \right) .$$

This is the free theory to which we add the interaction terms encoded in the 3- and 4-point functions. They are obtained as usual, with the help of the physical state condition $\mathsf{L}^a \mathsf{A}_a = 0$, by multiplication with ghost propagators and after summation over all permutations of the insertion points x_i. The result is

$$\begin{aligned}
\mathcal{S}^{(3)} &= -\frac{1}{2} \operatorname{tr} \left(\mathsf{L}_a \mathsf{A}_b * \left[\mathsf{A}^a \overset{*}{,} \mathsf{A}^b \right] + \left[\mathsf{A}_a \overset{*}{,} \mathsf{A}_b \right] * \mathsf{L}^a \mathsf{A}^b - \frac{2i}{3} \mathsf{f}^{abc} \mathsf{A}_a * \left[\mathsf{A}_b \overset{*}{,} \mathsf{A}_c \right] \right) , \\
\mathcal{S}^{(4)} &= -\frac{1}{4} \operatorname{tr} \left(\left[\mathsf{A}_a \overset{*}{,} \mathsf{A}_b \right] * \left[\mathsf{A}^a \overset{*}{,} \mathsf{A}^b \right] \right) .
\end{aligned}$$

Up to their normalization, the formulas for $\mathcal{S}^{(3)}$ and $\mathcal{S}^{(4)}$ are obvious from eq. (4.4) and our description of the leading contribution to the 4-point function. The relative normalization of $\mathcal{S}^{(3)}, \mathcal{S}^{(4)}$ and $\mathcal{S}_0$ may be determined most easily by comparison with

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the abelian case $\mathsf{f}_{abc} = 0$. The full effective action

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}^{(3)} + \mathcal{S}^{(4)} \tag{4.6}$$

for the scalar fields on our brane in $\mathbb{R}^7 \times S^3$ can be brought into a "geometric" form by introducing a field strength

$$\mathsf{F}_{ab}(\mathsf{A}) = i\mathsf{L}_a\mathsf{A}_b - i\mathsf{L}_b\mathsf{A}_a + i[\mathsf{A}_a \stackrel{*}{,} \mathsf{A}_b] + \mathsf{f}_{abc}\mathsf{A}^c \tag{4.7}$$

and a non-commutative analogue of the Chern-Simons form

$$\mathsf{C}S_{abc}(\mathsf{A}) = \mathsf{L}_a\mathsf{A}_b * \mathsf{A}_c + \frac{1}{3}\mathsf{A}_a * [\mathsf{A}_b \stackrel{*}{,} \mathsf{A}_c] - \frac{i}{2}\mathsf{f}_{abd}\mathsf{A}^d * \mathsf{A}_c \,. \tag{4.8}$$

Then our effective action can be written as sum of a Yang-Mills theory and of a Chern-Simons theory on the fuzzy sphere,

$$\mathcal{S} = \mathcal{S}_{\rm YM} + \mathcal{S}_{\rm CS} = \frac{1}{4}\,\mathrm{tr}\left(\mathsf{F}_{ab} * \mathsf{F}^{ab}\right) - \frac{i}{2}\,\mathrm{tr}\left(\mathsf{f}^{abc}\mathsf{C}S_{abc}\right). \tag{4.9}$$

In order to render the effective action dimensionless, the expression on the right-hand side is to be multiplied with the appropriate power $(2\pi\alpha')^2$ of the string tension.

In the computation leading from (4.6)–(4.9) we have added a term of second order in the fields A_a that vanishes on-shell, i.e. upon using the physical state condition $\mathsf{L}^a\mathsf{A}_a = 0$. The main motivation for this modification is that it renders the action off-shell gauge invariant. In fact, some cumbersome but straightforward manipulations show that

$$\delta_\Lambda \mathcal{S}_{\rm YM}(\mathsf{A}) = \delta_\Lambda \mathcal{S}_{\rm CS}(\mathsf{A}) = 0\,,$$

where

$$\delta_\Lambda \mathsf{A}_a = i\mathsf{L}_a\Lambda + i\,[\mathsf{A}_a, \Lambda]$$

for arbitrary $\Lambda \in \mathrm{Mat}(N) \otimes \mathrm{Mat}(2\alpha+1)$. Note, in particular, that the "mass term" in the Chern-Simons form (4.8) is required by gauge invariance. On the other hand, the effective action (4.9) is the unique combination of $\mathcal{S}_{\rm YM}$ and $\mathcal{S}_{\rm CS}$ in which mass terms cancel. As we shall see below, it is this special feature of our action that allows solutions describing translations of the branes on the group manifold. The action $\mathcal{S}_{\rm YM}$ was already considered in the non-commutative geometry literature [15, 14], where it was derived from a Connes spectral triple and viewed as describing Maxwell theory on the fuzzy sphere. Arbitrary linear combinations of non-commutative Yang-Mills and Chern-Simons terms were considered in [16].

There exists a nice way to rewrite our actions such that the statements about gauge invariance become completely obvious. To this end we note that the covariant derivative $\mathsf{L}_a + [\mathsf{A}_a, \cdot\,]$ can be implemented as a commutator with the element

$$\mathsf{B}_a = \mathsf{Y}_a + \mathsf{A}_a\,, \quad \text{where} \quad \mathsf{Y}_a = \frac{1}{\sqrt{2\alpha'}}\mathrm{Y}^1_a\,, \tag{4.10}$$

see eq. (3.6). The behavior of B_a under gauge transformations is given by $\delta_\Lambda B_a = i[B_a, \Lambda]$. This property guarantees the gauge invariance of $\mathcal{S}_{\rm CS}$ and $\mathcal{S}_{\rm YM}$ once we

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observe that both actions can be expressed entirely through B_a according to

$$\mathcal{S}_{\rm YM} = \frac{1}{4}\,\mathrm{tr}\left(\mathsf{F}_{ab} * \mathsf{F}^{ab}\right) \quad \text{with} \quad \mathsf{F}_{ab} = i[\mathsf{B}_a \stackrel{*}{,} \mathsf{B}_b] + \mathsf{f}_{abc}\mathsf{B}^c\,, \tag{4.11}$$

$$\mathcal{S}_{\rm CS} = -\frac{i}{2}\,\mathrm{tr}\left(\frac{1}{3}\mathsf{f}^{abc}\mathsf{B}_a * \mathsf{B}_b * \mathsf{B}_c - \frac{i}{\alpha'\mathsf{k}}\mathsf{B}_a * \mathsf{B}^a + \frac{i}{3\alpha'\mathsf{k}}\mathsf{Y}_a * \mathsf{Y}^a\right). \tag{4.12}$$

The two formulas are rather easy to verify by inserting the definition (4.10) of B_a and using the algebraic properties of Y_a. The full low-energy effective action for branes in S^3 then takes the simple form

$$\mathcal{S} = \mathrm{tr}\left(-\frac{1}{4}[\mathsf{B}_a \stackrel{*}{,} \mathsf{B}_b] * [\mathsf{B}^a \stackrel{*}{,} \mathsf{B}^b] + \frac{i}{3}\mathsf{f}^{abc}\mathsf{B}_a * [\mathsf{B}_b \stackrel{*}{,} \mathsf{B}_c] + \frac{1}{6\alpha'\mathsf{k}}\mathsf{Y}_a * \mathsf{Y}^a\right). \tag{4.13}$$

Up to now, we have only discussed SU(2) branes in bosonic string theory, which are unstable due to the presence of tachyonic modes. In the remainder of this section, we provide the formulas necessary to extend the bosonic discussion from above to the superstring case. As before, we will mostly ignore the curvature of the external manifold M^7 and pretend that our target has the form $\mathbb{R}^7 \times S^3$. In the supersymmetric situation, the SU(2) WZW model is supplemented by three free fermions $\psi^a(x)$, $a = 1, 2, 3$, with the following operator product expansions (we use CFT conventions as in eq. (3.2), string units are obtained by rescaling with α' in the same way as before):

$$\psi^a(x_1)\psi^b(x_2) = \frac{\mathsf{k}}{2}\frac{\delta^{ab}}{x_1 - x_2} + \cdots\,, \qquad J^a(x_1)\psi^b(x_2) = \frac{i f^{ab}{}_c}{x_1 - x_2}\psi^c(x_2)\,.$$

The world-sheet supercurrent is given by (see e.g. [26])

$$G = G^7 + G_{\rm SWZW} = G^7 + \frac{2}{\mathsf{k}}\delta_{ab}J^a\psi^b + \frac{4i}{3\mathsf{k}^2}f_{abc}\psi^a\psi^b\psi^c\,.$$

G^7 depends on the specific superstring background under consideration. e.g. for the NS 5-brane, we have [21]

$$G^7 = \eta_{\mu\nu}J^\mu\psi^\nu + J^6\psi^6 - \frac{2i}{\mathsf{k}}\partial\psi^6\,,$$

where $J^\mu = i\partial X^\mu$, ψ^μ for $\mu = 0, \ldots, 5$ are the free fields from the $\mathbb{R}^{5,1}$ subtheory and where $J^6 = i\partial\phi$ is formed from the Feigin-Fuchs boson in the linear-dilaton theory, with associated free fermion ψ^6. An explicit expression for G^7 in the $AdS_3 \times T^4$ model can be found e.g. in [27]; it is rather similar to the SU(2) supercurrent because of the relation of AdS_3 and the SU(1, 1) WZW model.

We are interested in BPS branes here, so in particular the world-sheet supercurrent must be preserved by the corresponding boundary conditions, i.e. we must have $G(z) = \pm\overline{G}(\bar{z})$ along the boundary of the worldsheet. It is easy to see that, in the NS 5-brane case, this enforces Neumann conditions on the linear dilaton: all the branes discussed here will end on the stack of NS 5-branes. As usual, the allowed number of Neumann directions in the remaining target dimensions depends on whether we work in IIA or IIB string theory.

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It is convenient to pass to a new set of SU(2) currents defined by

$$\tilde{J}^a := J^a + \frac{i}{\mathsf{k}} f^a{}_{bc} \psi^b \psi^c \,,$$

which satisfy the WZW operator product expansion (3.2) at level $\mathsf{k}-2$ instead of k, and which commute with the fermions ψ^a. (On AdS_3, one proceeds analogously for the currents of the SU(1,1) subtheory.) Using the new fields, it becomes clear that the CFTs under consideration split into tensor products of a bosonic part with ten free fermion, and that the same is true for the boundary states describing branes in the NS 5-brane or $AdS_3 \times S^3 \times T^4$ superstring background. As a consequence, space-time supersymmetry charges can be built up from the usual spin fields, and also the behavior under the GSO projection carries over from the flat target case. In particular, the partition functions of our supersymmetric branes in $M^7 \times S^3$ vanish, meaning that the spectra are tachyon-free and stable. Geometric arguments for stability of branes in S^3, based on the non-vanishing B-field and on the quantization condition on the brane labels α, were presented recently in [19, 20]. Our analysis of the equations of motion will, among other things, confirm these results.

Let us now collect formulas for the physical vertex operators necessary to compute the low-energy effective action in the supersymmetric case. Those corresponding to gauge bosons and scalars on the external part of the brane world-volume can be treated as usual, therefore we concentrate on the "Higgs scalars" associated with the S^3-directions. In the picture with superghost number -1, their vertex operators are $V_H^{(-1)}[\mathsf{A}](x) = V[e_k] : \psi^a V[\mathsf{A}_a] : (x)$ with corresponding states

$$\left| V_H^{(-1)}[\mathsf{A}] \right\rangle = a_a^{Jm} |k\rangle \otimes \psi^a_{-1/2} \left| Y^J_m \right\rangle .$$

Here, the SU(2) highest weight states are tacitly understood to contain the vacuum of the fermionic Fock space. Moreover, we now approximate the external part M^7 by flat space-time such that k is a $\tilde{p}$-momentum where $\tilde{p}$ is the number of Neumann directions in $\mathbb{R}^7$. As before, the parameters A_a are subject to the physical state condition $\mathsf{L}^a \mathsf{A}_a = 0$.

For the computation of 3- and 4-point functions, we also need vertex operators in the ghost number 0 picture. From the usual commutation relations between G_{SWZW} and ψ^a, J^a, one finds the SU(2) component of the corresponding states to be

$$\begin{aligned} \left| V_H^{(0)}[\mathsf{A}] \right\rangle &= G_{-1/2} \left| V_H^{(-1)}[\mathsf{A}] \right\rangle = \\ &= a_a^{Jm} \left(\tilde{J}^a_{-1} - \frac{2}{\mathsf{k}} \delta_{cb} \tilde{J}^c_0 \psi^a_{-1/2} \psi^b_{-1/2} - \frac{i}{\mathsf{k}} f^a{}_{bc} \psi^b_{-1/2} \psi^c_{-1/2} \right) \left| Y^J_m \right\rangle . \end{aligned}$$

It is now straightforward to compute the 3-point function

$$\left\langle V_H^{(-1)}[\mathsf{A}_1](x_1) V_H^{(-1)}[\mathsf{A}_2](x_2) V_H^{(0)}[\mathsf{A}_3](x_3) \right\rangle$$

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and the result is given by the terms displayed in eq. (4.4). In the supersymmetric case, there are no higher order corrections in α'. Likewise, the leading contribution to the 4-point function is as in the bosonic WZW model.

All in all, the low-energy effective action (4.9), supplemented by the non-abelian Yang-Mills-Higgs theory for the external degrees of freedom, also describes the dynamics of supersymmetric branes in $\mathbb{R}^7 \times S^3$. Of course, there are additional terms we did not discuss, which mix the fields from the external-gauge theory with the Higgs scalars from the S^3 factor. To leading order in α', they can be copied from the effective action for flat branes in $\mathbb{R}^{10}$. We will not discuss space-time fermions here, whose action could in principle be computed from world-sheet Ramond fields, or by exploiting space-time supersymmetry.

5. Classical solutions and bound states

Having found the action that describes the dynamics of branes on SU(2), we can start analyzing the classical equations of motions and their solutions. The equations of motion which derive from the action (4.9) read

$$\mathsf{L}^a \mathsf{F}_{ab} + [\mathsf{A}^a \stackrel{*}{,} \mathsf{F}_{ab}] = 0 \,. \tag{5.1}$$

Geometrically, this means that the curvature of the solutions must be covariantly constant. When we deal with a stack of N branes which wrap the fuzzy sphere labeled α, the equations are to be read as equations in $\mathrm{Mat}(N) \otimes \mathrm{Mat}(2\alpha+1)$, and $\mathsf{L}^a = 1 \otimes \mathsf{L}^a$ acts trivially on the Chan-Paton factors.

We will first concentrate on constant solutions and then extend our discussion to non-constant ones towards the end of the section. On constant fields S_a, i.e. those satisfying $L^a \mathsf{S}_b = 0$ for all a, b, the equations of motion (5.1) simplify to

$$\Big[\mathsf{S}^a \stackrel{*}{,} [\mathsf{S}_a \stackrel{*}{,} \mathsf{S}_b] - i\, \mathsf{f}_{abc}\, \mathsf{S}^c\Big] = 0 \,. \tag{5.2}$$

As we will see in a moment, even the constant solutions give rise to a rather non-trivial pattern of D-brane configurations. The equations (5.2) possess two different types of solutions. Obviously, they are solved by any set of commuting matrices $\mathsf{S}_a \in \mathrm{Mat}(N) \otimes \mathbf{1}_{2\alpha+1}$. These solutions correspond to marginal fields in the SU(2) theory which translate our N D-branes in the group target, but they have no influence on the radius. The new positions of the branes are determined by the eigenvalues of the matrices S_a. If some of these eigenvalues agree, smaller stacks of spherical branes appear in the final configuration. All these statements are familiar from D-branes in flat space [28] and they can be verified rigorously along the lines of [29].

Remarkably, there exists a second type of constant solutions without any flat space analogue. They are given by constant flat connections, or equivalently, by matrices $\mathsf{S}_a \in \mathrm{Mat}(N) \otimes \mathbf{1}_{2\alpha+1}$ satisfying the commutation relations

$$[\mathsf{S}_a, \mathsf{S}_b] = i\mathsf{f}_{ab}{}^c \mathsf{S}_c \tag{5.3}$$

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which, on constant solutions, are equivalent to $\mathsf{F}_{ab}(\mathsf{S}) = 0$. Equations (5.3) simply characterize N-dimensional representations of su(2). To any such representation π_N we can assign a partition $(n_i)_{i=1,\dots,r}$ of $N = n_1 + \cdots + n_r$ by choosing n_i to be the dimensions of the irreducible subrepresentations in π_N. This partition characterizes the original representation uniquely up to equivalence, hence it provides a simple classification for solutions of the second type. Our objective is to identify these solutions with known configurations of branes. Rigorous statements about this identification are difficult since the boundary perturbations generated by the set S_a are marginally relevant. This implies that there is no continuous trajectory in the moduli space of conformal field theories that we could follow. In this respect, the same problems arise in the study of marginally relevant operators as with open string tachyons and their condensation [29]–[32]. Nevertheless, we will provide strong evidence for the pattern of renormalization group fixed points we are about to suggest.

To this end, recall that the state space of our boundary theory is given by $\mathrm{Mat}(N) \otimes \mathcal{H}_\alpha$ where $\mathcal{H}_\alpha$ is the state space of a single brane with label α, and $\mathrm{Mat}(N)$ are the wave functions associated with the Chan-Paton factors. The modes j^a_n of the chiral fields $j^a(z)$ generate an action of $\widehat{\mathrm{SU}}(2)_{\mathsf{k}}$ on the second-tensor factor. Moreover, there is an action of su(2) coming with our solution S_a. It is defined through the commutator of S_a with the Chan-Paton matrices in the first tensor factor $\mathrm{Mat}(N)$ of our state space, i.e. S_a act as $\mathrm{ad}(\mathsf{S}_a)$. Once we turn on the perturbation

$$S_{\mathrm{pert}} = \int dx : j^a V[\mathsf{S}_a] : (x) = \int dx\, j^a(x) \mathsf{S}_a \,,$$

these two "symmetries" are broken because the perturbing operator couples the $\mathrm{SU}(2)$ current to the quantum mechanical su(2) of the Chan-Paton factors. From experience with similar couplings of spin and orbital angular momentum in quantum mechanics, one may suspect that only the sum $j^a + \mathsf{S}^a$ has a good chance to be realized as a symmetry in the perturbed theory. Indeed, the fact that our matrices S_a obey the equations of motion (5.3) implies that the sum $j^a_n + \mathrm{ad}(\mathsf{S}^a)$ gives rise to an action of the current algebra $\widehat{\mathrm{SU}}(2)_{\mathsf{k}}$ with level k on the state space $\mathrm{Mat}(N) \otimes \mathcal{H}_\alpha$. Hence, if we assume that the action survives the perturbation, we can use it to determine the representations of $\widehat{\mathrm{SU}}(2)_{\mathsf{k}}$ that are present in the perturbed theory. The decomposition is easy to find with the help of zero modes, and it turns out that there is a unique way of combining their characters back into partition functions of the known D-brane theories. In principle, the partition functions do not quite suffice to recover all information about the D-branes since they are invariant under translations on the background. However, the position of the branes can be detected by bulk fields [33], which are sensitive only to the scalar part of the Chan-Paton factors. The fact that $\mathrm{tr}\, \mathsf{S}_a$ vanishes for all solutions of eqs. (5.3), shows that, to first order in the perturbative expansion, the branes are not translated, i.e. their classical counterparts are conjugacy classes centered around the origin e, as listed after eq. (3.1).

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The above analysis leads to the following scenario: when we start from a stack of N α-branes centered around e, the solution S_a of eqs. (5.3) corresponds to the fixed point of an RG flow that takes us to a new brane configuration consisting of a superposition of spherical branes, again centered around e. These branes can have different radii, and they can be stacked or just single branes. To be more precise, we use the associated partition (n_i) and introduce spins $J_i = (n_i - 1)/2$. Then the boundary state for the final D-brane configuration can be written as sum of "elementary" boundary states $|\beta\rangle$ for single spherical branes,

$$|(\mathsf{S}_a)\rangle = \sum_\beta N_\beta |\beta\rangle \qquad \text{with} \qquad N_\beta = \sum_i N^\beta_{\alpha J_i} \,. \tag{5.4}$$

Here, $N^\beta_{\alpha J}$ denote the fusion rules of su(2). The multiplicity N_β is the size of the stack of branes of type β contained in the final configuration.

As an example, let us consider an initial stack of N D0-branes at the origin and choose an irreducible N-dimensional representation S_a of su(2). Then, according to our rules, this will decay into a single brane of type $\beta = (N-1)/2$. In other words, the stack of D0-branes has evolved into a single brane that wraps a non-degenerate 2-dimensional (fuzzy) sphere. One could say that, in this way, we have dynamically created two new dimensions on the expense of breaking the gauge symmetry from U(N) to U(1). Had we used some reducible representation S_a instead, with partition $(n_i)_{i=1,\dots,r}$, stacks of $N_\beta = \#\{n_i \mid n_i = 2\beta + 1\}$ branes would have appeared, which wrap the spheres with labels β. In case there were degeneracies among the n_i, the gauge group would be only partially broken to $\bigtimes_\beta U(N_\beta)$.

We want to use the example of N D0-branes blowing up into a single D2-brane to provide more evidence for our identification of the fixed points. The idea is to exploit that we certainly know the tension of all the maximally symmetric branes. If our identification is correct, the leading term in the difference of tensions between the initial and the final brane configuration has to agree with the effective action $\mathcal{S}(\mathsf{S}_c)$ evaluated at the classical solution. The strategy is similar to the one used in [34].

The tension of a single brane is essentially given by the logarithm of the so-called g-factor of the boundary conformal field theory, see e.g. [35, 36]. The latter is defined as the 1-point function of the identity operator [37], and for a brane of type α in the SU(2) WZW model it is given by [24]

$$g_\alpha = \frac{S_{\alpha 0}}{\sqrt{S_{0\,0}}} = \left(\frac{2}{\mathsf{k}+2}\right)^{1/4} \frac{\sin\frac{(2\alpha+1)\pi}{\mathsf{k}+2}}{\sin^{1/2}\frac{\pi}{\mathsf{k}+2}} \,.$$

Here S_{IJ} denotes the modular S-matrix of the WZW model. For the initial stack of N identical D0-branes, the g-factor is $g_{N\,\mathrm{D0}} = N\, g_0$. If we expand the difference of

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the logarithms in a power series around $1/\mathsf{k} = 0$ we find to leading order

$$\ln \frac{g_{(N-1)/2}}{g_{N\,\mathrm{D0}}} = -\frac{\pi^2}{6}\frac{N^2-1}{\mathsf{k}^2}\,.$$

The minus sign shows that the tension of the final state is lower than the one of the original stack of branes, in agreement with the "g-theorem" stating that the g-factor should always decrease along the renormalization group trajectory [37].

Now we have to compare our result on the g-factors with the value of the action $\mathcal{S}(\mathsf{S}_a)$ where S_a is a solution of eqs. (5.3) corresponding to an irreducible N-dimensional representation of su(2). Because S_a is constant, all the terms in which L_b acts on S_a vanish, and after a short computation we obtain

$$\mathcal{S}(\mathsf{S}_a) = -\frac{(2\pi\alpha')^2}{6\alpha'\mathsf{k}}\,\mathrm{tr}(\mathsf{S}_c * \mathsf{S}^c)\,.$$

Up to normalization, the term in the argument of the trace is simply the Casimir element of su(2) evaluated in the irreducible representation of spin $\alpha = (N-1)/2$, i.e. the argument of the trace is proportional to $\alpha(\alpha+1) = (N+1)(N-1)/4$. Taking the normalization into account, we find

$$\mathcal{S}(\mathsf{S}_a) = \ln \frac{g_{(N-1)/2}}{g_{N\,\mathrm{D0}}}$$

as desired. Note that the value of the action (the potential energy of the Higgs fields) at the new fixed point is actually lower than in the original state which means that the N D0-branes have formed a bound state.

So far we have only discussed constant solutions to the full equations of motion (5.1). For D0-branes, this discussion is complete since any field A_a on a single point is constant. On the other hand, we have obtained all other branes α as bound states of D0-branes. Combining the two facts, it should be possible to understand non-constant classical solutions, as well. To this end, we first rewrite the equation of motion (5.1) according to

$$\Big[\mathsf{B}^a \overset{*}{,} [\mathsf{B}_a * \mathsf{B}_b] - i\mathsf{f}_{abc}\mathsf{B}^c\Big] = 0 \tag{5.5}$$

with $\mathsf{B}_a = \mathsf{Y}_a + \mathsf{A}_a$ as before. Now the general equations of motion (5.1) have become formally identical to (5.2) which govern the dynamics of D0-branes, and solving them amounts to finding matrices $\mathsf{B}_a \in \mathrm{Mat}(N) \otimes \mathrm{Mat}(2\alpha+1)$ that satisfy our familiar eqs. (5.5). From any such solution B_a we can then obtain a field $\mathsf{A}_a = \mathsf{B}_a - \mathsf{Y}_a$ that will carry us into a new RG-fixed point upon perturbation. Hence, it only remains to identify this fixed point. In case that B_a is a representation of su(2), we claim that the fixed point is determined by the partition (n_i) characterizing the representation B_a of su(2) along with the rules we have used for stacks of D0-branes above. This means that there will appear $N_\beta = \#\{\, n_i \,|\, n_i = 2\beta+1\}$ branes wrapping the spheres with labels β in the final configuration.

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Note that this pattern is consistent with the one proposed in (5.4). In fact, if $\mathsf{A}_a = \mathsf{S}_a$ is constant, i.e. if it commutes with Y_a, then the elements B_a generate the tensor product of the $(2\alpha+1)$-dimensional irreducible representation with the representation S_a. Hence, the partition associated with B_a can be obtained from the one that comes with A_a upon fusing with the spin α representation.

Our claim on the nature of the fixed points described by su(2) representations B_a can be substantiated with the same methods as before. In particular, one can again compare the g-factors of the proposed fixed points with the value of the effective action. When evaluated on a solution of our equation of motion (5.5), the latter becomes

$$\mathcal{S}(\mathsf{B}_a) = \frac{1}{12}\,\mathrm{tr}\left([\mathsf{B}_a * \mathsf{B}_b] * \left[\mathsf{B}^a * \mathsf{B}^b\right] + \frac{2}{\alpha'\mathsf{k}}\mathsf{Y}_a * \mathsf{Y}^a\right). \tag{5.6}$$

The formula shows that commuting matrices B_a cause the action to increase, i.e. they lead to a final configuration which has a larger mass than the one we started with. On the other hand, the lowest action among all the solutions we have described is assumed for B_a being an irreducible representation of su(2). This corresponds to the stack of branes merging into a single brane with $\beta = N\alpha + (N-1)/2$.

In addition, one may compute the second variation of the action and evaluate it at an $N(2\alpha+1)$-dimensional su(2) representation B_a to get

$$\delta^2\mathcal{S} = \mathrm{tr}\Big(-[\mathsf{B}_a \stackrel{*}{,} \delta\mathsf{B}_b] * \left[\mathsf{B}^a \stackrel{*}{,} \delta\mathsf{B}^b\right] + [\mathsf{B}_a \stackrel{*}{,} \delta\mathsf{B}^a]\left[\mathsf{B}_b \stackrel{*}{,} \delta\mathsf{B}^b\right]\Big).$$

Using the fact that the map $\mathsf{A} \mapsto [\mathsf{B}_a, [\mathsf{B}^a, \mathsf{A}]]$ is invertible on all traceless matrices A, one can easily see that the su(2) representations B_a represent a local minimum of the action (except from the obvious zero modes associated with rigid translations), in agreement with the results of [19, 20] on the stability of the spherical branes in WZW models.

The previous analysis of non-constant solutions may be turned around and used to "derive" the equations of motion (5.1) for an arbitrary brane of type α from those of D0-branes.[1] In fact, we may think of the fields A_a on the α-brane as fluctuations around the constant solution S_a which takes us from N D0-branes to the single D2-brane. Now we recall that S_a is an irreducible $(2\alpha+1)$-dimensional representation of su(2). The world-volume algebra of the α-brane, on the other hand, contains the elements Y_a which also generate an irreducible su(2) representation of dimension $2\alpha+1$. Since any two irreducible su(2) representation of the same dimension are equivalent, we can identify S_a with Y_a so that the idea of studying small fluctuations around S_a motivates to insert the combination $\mathsf{Y}_a + \mathsf{A}_a$ into the equations (5.2). The resulting equations are given by (5.5) and do indeed describe the full dynamics of the α-brane.

Let us finally note that our discussion of the constant solutions coincides with the "absorption of boundary spin" that has appeared in the work of Affleck and Ludwig on the multichannel Kondo-problem [37]. In that context, an irreducible representa-

[1] This argument was suggested to us by J. Maldacena.

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tion $\mathsf{S}_a \in \mathrm{Mat}(N)$ is reinterpreted as the quantum mechanical spin variable of a spin $J = (N-1)/2$ impurity. According to Affleck and Ludwig, this system evolves into a new low-temperature fixed point in which the boundary spin gets absorbed.

6. Conclusions and outlook

In this paper we have studied the dynamics of branes in $M^7 \times S^3$ which, classically, wrap some $S^2 \subset S^3$, focussing on the limit where the radius $R \sim \sqrt{\alpha' \mathsf{k}}$ of S^3 grows very large. The fact that the spherical conjugacy classes become fuzzy spheres in the quantum theory has a number of consequences for the low-energy effective action.

First, the gauge fields on the external part of the branes become non abelian already on a single brane. The size of the gauge group $\mathrm{U}(N(2\alpha+1))$ depends on the number N of coinciding branes and on the label α of the fuzzy sphere that is wrapped in S^3.

Second, the Higgs potential for the scalars associated with the internal S^3 consists of a non-commutative Yang-Mills theory on the fuzzy sphere plus a non-commutative analogue of the Chern-Simons action. As expected on general grounds, this LEEA enjoys invariance under non-commutative gauge transformations.

We have studied solutions to the equations of motion derived from the non-commutative part of the effective action. We found that a certain class of constant field configurations, namely those obtained from pairwise commuting constant gauge fields, are associated with moduli that trigger rigid translations of branes in the S^3 background. The more interesting type of solutions is given by reducible or irreducible representations of su(2) and has no analogue for flat branes in $\mathbb{R}^d$. These solutions describe fixed points of RG flows driven by marginally relevant perturbations of the world-sheet theory. At the fixed points, the initial stack of identical branes has evolved into a bound state of SU(2) branes, whose precise form is determined by the su(2) representation. In contrast to the "constant commuting case", the new type of solutions given in (5.3) can change the radii of the branes. In this way, we can in particular interpret 2-dimensional spherical branes as bound states formed from a stack of D0-branes.

As we pointed out above, gauge theories on fuzzy spheres were discussed in the literature before [12]–[16], and the field equations that were derived from such actions are very similar to the equations we have studied here. The non-commuting solutions (5.3) are a common feature of these cases, but the model we have derived here from perturbative string theory is distinguished by the property that it also admits the commuting solutions corresponding to rigid movements in S^3.

Based on what we have described above, one cannot exclude the existence of additional solutions which could lead the system into new types of bound states. Even if such configurations exist, it is likely that the single spherical brane realizes the global minimum of the action. We have not been able to prove this in the general case but at least it is true for $N(2\alpha+1) = 2$.

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Extending the present analysis to finite radius $R \sim \sqrt{\alpha' \mathsf{k}}$ is another challenge. From the results of [7] one may be tempted to believe that part of the picture could survive if the Lie algebra su(2) is replaced by the corresponding quantum group and its representation theory. The relation with quantum group representations was also stressed in [38]; their importance also transpires from the general investigations in [33, 39, 40]. Note that in the Kondo problem, the level k counts the number of conduction bands. Hence, Kondo physics explores the region of small level. In this context one may recall that for $N = \mathsf{k}n/2$ and n integer, which of course requires to go beyond a perturbative analysis in $1/\mathsf{k}$, the known fixed-points are described by free rather than interacting theories [41]. This suggests that new phenomena occur for stacks of D0-branes on a sphere of finite size. For instance, the analysis of Affleck and Ludwig [41] shows that a stack of N D0-branes can evolve into a single spherical brane with label α where $\alpha \leq \mathsf{k}/2$ is determined from N by the rule $N = \mathsf{k}n/2 + \alpha$. In view of the work [42], which compares "giant gravitons" with spherical branes on SU(2), such finite level effects should be related to the "stringy exclusion principle".

As we have pointed out before, most of our statements admit obvious generalizations to other group manifolds. The necessary CFT ingredients can be recovered e.g. from [33, 19]. In view of the perturbative treatment of [5] and of the general definition of non-commutative world-volumes from boundary CFT given in [7], we believe that the lessons we have learned are much more universal. Flows from stacks of D0-branes to branes whose classical counterparts have higher-dimensional world-volume should be a common phenomenon in string compactifications. It is a distinguished feature of string theory that the world-sheet perspective provides natural descriptions of effects that appear like a "topology change" in the space-time picture, e.g. the dynamical generation of extra world-volume dimensions as discussed above, or the rotation of Dirichlet into Neumann conditions for free bosons by marginal operators [29]. We hope that extensions of our results may also provide some quantitative insight into tachyon condensation and bound state formation on other more complicated string backgrounds which possess an exact conformal field theory description. This applies especially to Gepner models and their associated D-brane theories [36, 43]. Another step in this direction was taken recently in [34], and we hope to return to these issues in the future.

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